

# De-leptonization and Non–Axisymmetric Instabilities in Core Collapse Supernovae

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## ABSTRACT

The timescale of de-leptonization by neutrino loss and associated contraction of a proto-neutron star is short compared to the time to propagate a shock through the helium core of a massive star, and so the de-leptonization phase does not occur in the vacuum of space, but within the supernova ambiance whether or not there has been a successful explosion. Dynamical non–axisymmetric instabilities (NAXI) are predicted for sufficiently strongly differentially rotating proto–neutron stars. Some modes are unstable for small values of the ratio of rotational kinetic energy to binding energy,  $T/|W| \gtrsim 0.01$ . The NAXI are likely to drive magnetoacoustic waves into the surrounding time–dependent density structure. These waves represent a mechanism of the dissipation of the free energy of differential rotation of the proto–neutron star, and the outward deposition of this energy may play a role in the supernova explosion process. We estimate the power produced by this process and the associated timescale and discuss the possible systematics of the de-leptonization phase in this context. A likely possibility is that the proto-neutron star will spin down through these effects before de-leptonization and produce substantial but not excessive energy input.

*Subject headings:* hydrodynamics – instabilities – stars: rotation – stars: magnetic fields – stars: neutron – supernovae: general

## 1. Introduction

Core collapse of massive stars affects a broad range of astrophysics including the production of neutron stars and black holes, the creation and dissemination of heavy elements, and the potential production of gravitational radiation, but the physics of core collapse supernovae is not yet fully understood. Neutrino processes play a major role and asymmetries are ubiquitous (Wang et al. 1996, 2001; Leonard et al. 2006).

After its birth as a hot, rapidly spinning proto-neutrons star (PNS) at core bounce, a neutron star that avoids being converted to a black hole will de-leptonize, shrink in radius, and spin up. The evolution in this phase, particularly in the presence of magnetic fields, has not been thoroughly explored, but is critical to establish the rotation and magnetic field distribution of the neutron star as it becomes accessible to observation in the center of the explosion. One of the processes that could enhance the depletion of the angular momentum of the neutron star is the development of non-axisymmetric instabilities (NAXI) with the attendant generation of magnetoacoustic waves. When differential rotation is strong, the value of the ratio of rotational kinetic energy to binding energy,  $T/|W|$ , for onset of NAXI can be rather low (see §2). If the threshold for NAXI is low, these instabilities may be relevant to a significant range of rotating core-collapse situations. Much of the literature on NAXI in neutron stars ignores the hot PNS conditions just after core collapse, magnetic fields, and the time-dependent density structure that surrounds the new-born neutron star until this matter is swept away in the supernova explosion. These complications may have a major impact on the rotational and magnetic state of the surviving, observable neutron star. We explore some of the possible effects here.

Section 2 gives a brief summary of the literature of non-axisymmetric instabilities of differentially rotating neutron stars and an estimate of the rate of dissipation of rotation by NAXI. We consider the de-leptonization phase in §3. Conclusions are given in §4.

## 2. Non-Axisymmetric Modes in Neutron Stars

For sufficiently large values of  $T/|W| \gtrsim 0.27$ , rotating incompressible spheroids will be dynamically unstable to formation of a rotating bar configuration (Chandrasekhar 1969). For  $T/|W| \gtrsim 0.14$  dissipation leads to a secular instability and again to a bar-like mode (Chandrasekhar 1969; Shapiro 2004; Ou et al. 2004). For compressible, differentially rotating configurations, the limit for dynamical instability can be decreased (Tohline & Hachisu 1990; Rampp et al. 1998; Centrella et al. 2001; Imamura & Durisen 2004; Shibata & Sekiguchi 2005). Shibata et al. (2003) find that non-axisymmetric dynamical instabilities can occur for  $T/|W|$  as small as 0.01. This result was obtained in the absence of detailed microphysics that might result in damping of the instability, but nevertheless indicates the possible range of conditions for which such instabilities might occur. Watts et al. (2005) propose that differentially rotating stars have a continuous spectrum of modes with dynamical behavior that is distinct from discrete normal modes in uniformly rotating stars. They explain this behavior with the idea of resonances in a “co-rotation band.” Ott et al. (2005) found spiral wave instabilities of  $m = 1, 2$  modes with  $T/|W| = 0.08$ . Ou and Tohline (2006) explored

the growth of these modes and their dependence on the presence of a “cavity” (Lovelace et al. 1999; Li et al. 2000, 2001) and co-rotation point.

Little of the work on non-axisymmetric instabilities summarized here includes magnetic fields. Akiyama et al. (2003) (see also Thompson et al. 2005; Wilson et al. 2005; Blackman et al. 2006) showed that the MRI is likely to build up a strong, primarily toroidal, magnetic field on a time scale of tens of milliseconds. This conclusion presumes that the shear is not damped by other viscosities that would prevent the MRI altogether. This assumption must be checked in the future by detailed numerical studies. In the extreme case of a collapse that produces  $T/|W| \gtrsim 0.20$  at bounce it is likely that a dynamically unstable bar will be dissipated by shock and sound waves after several rotations and before a strong magnetic field can be generated by the MRI. If the initial value of  $T/|W|$  in the PNS is  $0.14 \lesssim T/|W| \lesssim 0.20$ , the magnetic field is expected to grow more rapidly than the attendant secular instability. In the context of a PNS, the dissipation that drives the secular formation of a bar is likely to be internal viscous dissipation rather than gravitational radiation reaction forces. An estimate of the dissipation time due to magnetic shear viscosity by Thompson et al. (2005) gives  $\tau_{diss} \sim (\alpha\Omega)^{-1}(r/h_P)^2$  where  $h_P$  is the pressure scale height and  $\alpha$  is a dimensionless parameter of order unity. From their numerical models, Thompson et al. (2005) find  $\tau_{diss} \sim 0.1 - 1$  s, much shorter than would be given by either neutrino viscosity or gravitational radiation reaction (Ou et al. 2004). This estimate, while plausible, does depend on this particular  $\alpha$  parameter being of order unity. The secular bars thus formed would be Jacobi-like with minimal internal flow, but a rotating pattern that would interact with the surrounding medium in which the bar rotates. The general instability is not expected to depend on the manner in which viscous heat is dissipated, but the specific configuration may do so (Shapiro 2004). Rampp et al. (1998) note that a bar-like structure will generate sound and shock waves like a “twirling-stick” and stress that collapse is not an equilibrium situation in terms of the instability and growth of non-axisymmetric modes. In what follows we assume that secular bars form by internal viscosity and dissipate by NAXI more slowly than the magnetic field saturates, but much faster than the de-leptonization timescale.

A saturation time for the magnetic field of tens of milliseconds is substantially longer than a dynamical time, but short compared to the saturation time of many of the relevant non-axisymmetric modes  $\sim 50 - 100$  ms (Shibata et al. 2003; Ott et al. 2005; Ou and Tohline 2006). In practice the instability to the MRI and those to NAXI may interact in a complex way (Rezzolla et al. 2000, 2001a,b). That is a very interesting issue, but beyond the scope of this paper. Here we will assume that a strong magnetic field does not inhibit the growth of at least some of the NAXI, but that the field is established prior to the saturation of all the non-axisymmetric modes except the dynamical bar modes.

Another critical time scale in this problem is that for the success of a supernova explosion. Given the failure of prompt explosions in state of the art model models, the characteristic growth times of non-axisymmetric modes are long compared to the time for the bounce shock to stall, 10s of ms, but short compared to the time for a successful shock to propagate out of the infalling iron core (seconds) and the helium core ( $\sim 10$  seconds) and the time to de-leptonize and contract ( $\sim 1 - 10$  s). If an explosion due to other effects (e.g., neutrino heating) precedes the non-axisymmetric effects, it cannot be very far along when the PNS becomes non-axisymmetric. In trial numerical calculations for which an explosion is induced (to be reported elsewhere), we do not find that the density profile is substantially altered on time scales of several hundred milliseconds, whether or not an explosion has ensued at that time. The formation and dissipation of non-axisymmetric modes in a strongly magnetic PNS thus may enhance an explosion that is already underway.

Any non-axisymmetric mode will tend to form sound waves, shock waves and Alfvén and other MHD waves by interaction with the surrounding, time-dependent, medium. The typical non-axisymmetric mode formed at modest rotation may be the  $m = 1$  spiral mode discussed by Ott et al. (2005) and Ou and Tohline (2006). While the details will surely differ, we approximate this spiral-like oscillation by the radial oscillation of a sphere that has a sinusoidal displacement of amplitude  $\delta r$ . Following Landau and Lifshitz (1959), we write for the magneto-acoustic luminosity:

$$L_{mhd} = 2\pi\rho v_{fast} \left(\frac{\delta r}{r}\right)^2 \frac{\omega^4 r^6}{(v_{fast}^2 + \omega^2 r^2)}, \quad (1)$$

where we have taken the velocity amplitude of the oscillations to be  $\delta v = \omega \delta r$  and wave number  $k = \omega/v_{fast}$  with  $\omega$  the angular frequency of the radiated magnetosonic waves. The phase velocity of fast magnetosonic waves is  $v_{fast}^2 = v_a^2 + c_s^2 \sim c_s^2$  for the case of sub-Keplerian rotation, relatively weak saturation fields, and hence  $v_a < c_s$ . In this limit the power radiated into fast magnetosonic waves and that into ordinary sonic waves is virtually the same. The flux into Alfvén waves, per se, will be substantially less than into sound and fast magnetosonic waves. There are (at least) three characteristic frequencies involved in this problem, the frequency of the radiated waves, the frequency of the unstable NAXI, and the rotational frequency. Instability arguments based on “co-rotation bands” suggest that the latter two are comparable. Here we will assume that all three frequencies are comparable and specifically make the identification that the frequency of the radiated waves is comparable to the rotational frequency, hence  $\omega \sim \Omega$ .

With moment of inertia  $I = 2/5 f_I M R^2$  and binding energy  $|W| = 3/5 f_W G M^2 / R$  where

$f_I$  and  $f_W$  are numerical factors of order unity we can write

$$\frac{T}{|W|} = \frac{1}{3} \frac{f_I}{f_W} \frac{R^3 \Omega^3}{GM}. \quad (2)$$

Eqn. 1 can then be expressed as:

$$\begin{aligned} L_{mhd} &= \frac{18\pi\rho G^2 M^2}{c_s} \left( \frac{f_W}{f_I} \frac{T}{|W|} \frac{\delta r}{r} \right)^2 \left[ 1 + \left( \frac{v_{rot}}{c_s} \right)^2 \right]^{-1} \\ &\sim 2.5 \times 10^{56} \text{ergs}^{-1} \rho_{12} c_{s,9}^{-1} M_{33}^2 \left( \frac{T}{|W|} \right)^2 \left( \frac{\delta r}{r} \right)^2 \left[ 1 + \left( \frac{v_{rot}}{c_s} \right)^2 \right]^{-1}, \end{aligned} \quad (3)$$

where the subscripts represent normalization in cgs units,  $v_{rot} = \Omega r$  and we expect  $v_{rot} \lesssim c_s$ .

The timescale for dissipation of the rotational kinetic energy by this MHD flux is then:

$$\begin{aligned} \tau_{mhd} &\sim \frac{\frac{1}{2} I \Omega^2}{L_{mhd}} = \frac{c_s}{30\pi\rho R G} \left( \frac{f_I^2}{f_W} \right) \left( \frac{T}{|W|} \right)^{-1} \left( \frac{\delta r}{r} \right)^{-2} \left[ 1 + \left( \frac{v_{rot}}{c_s} \right)^2 \right] \\ &\sim 1.6 \times 10^{-4} \text{s} \frac{c_{s,9}}{\rho_{12} R_6} \left( \frac{f_I^2}{f_W} \right) \left( \frac{T}{|W|} \right)^{-1} \left( \frac{\delta r}{r} \right)^{-2} \left[ 1 + \left( \frac{v_{rot}}{c_s} \right)^2 \right] \end{aligned} \quad (4)$$

These results are sensitive to specific parameters, especially the radius and the ambient density that will vary with position and time, the value of  $T/|W|$ , and the amplitude of the oscillation. Our choice of normalization was guided by the density and radius at which the shear and magnetic field are maximum and the amplitude by numerical simulations (Ott et al. 2005)<sup>1</sup>. With this choice of normalization we find that for a PNS of radius 50 km the timescale could be rather short even for rather small radial perturbations. Note that  $T/|W|$  is bounded by 0.01 and 0.14 in the framework we address here. For  $T/|W| \sim 0.01$ ,  $R_6 \sim 5$  and  $\delta r/r \sim 0.1$ , Eqn. 3 gives  $L_{mhd} \sim 2.5 \times 10^{50} \text{ erg s}^{-1}$  and Eqn. 4 gives  $\tau_{mhd} \sim 300 \text{ ms}$ , somewhat shorter than the de-leptonization time. For the same parameters, but with  $T/|W|$  near the upper limit, the luminosity would be 200 times larger and quite competitive with neutrino heating rates. Note also that these expressions are formally independent of the magnetic field (which enters through  $v_{fast}$ ), but that for realistic cases with strong gradients in magnetic field and perhaps concentrations of the field on the rotational axis, the exact nature of the dependence on the field strength and distribution could be rather more direct. Some of this power could tend to go out the equatorial plane, but significant power could be channeled up the rotation axis.

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<sup>1</sup>see <http://www.aei.mpg.de/cott/rotinst>

In summary, the ordering of time scales that will affect the physics of supernovae is roughly:

$$\tau_{dyn} < \tau_{rot} < \tau_{dyn-bar} < \tau_{shock} \sim \tau_{MRI} < \tau_{NAXI} \sim \tau_{mhd} \sim \tau_{secular} < \tau_{delep} < \tau_{explosion} \ll \tau_{grr} \quad (5)$$

where  $\tau_{dyn} \sim 1$  ms is the dynamical time scale,  $\tau_{rot}$  is the (sub-Keplerian) rotation timescale,  $\tau_{dyn-bar}$ , a few rotation times, is the timescale for dynamical bar formation,  $\tau_{shock} \sim 10$  ms is the time for the shock to form and stall,  $\tau_{MRI} \sim 30$  ms is the time for the MRI to grow the magnetic field to saturation,  $\tau_{NAXI} \sim 50 - 100$  ms is the time for non-axisymmetric ( $m = 1$ ) modes to grow,  $\tau_{mhd} \sim 0.1 - 1$  s is the time to spin down due to magnetosonic luminosity,  $\tau_{secular} \sim 0.1 - 1$  s is the time for a secular bar mode to grow,  $\tau_{delep} \sim 1 - 10$  s is the de-leptonization time of the PNS to contract to form a neutron star,  $\tau_{explosion} \sim 10$  s is the time for the successful shock to propagate out of the infalling iron core and into the surrounding star, e. g., the helium core, and  $\tau_{grr}$  is the time to dissipate angular momentum and rotational energy of the core by gravitational radiation reaction forces. In the current context,  $\tau_{explosion}$  is meant to be the time beyond which physical processes in the PNS will no longer affect the ultimate outcome, perhaps because the density has decreased sufficiently that even if magnetoacoustic flux is liberated, it cannot propagate outward and hence affect the explosion. In the absence of a detailed model of this process, we have taken the time for a successful shock to propagate into the helium core as a representative measure of this scale. We note that the epoch of the onset of convection within the PNS is of order of tens of ms and that of convection in the post-shock region is of order 100 ms. This emphasizes that timescales of order 0.1 to 1 s, not epochs of traditional concentration, will involve a variety of interacting physical processes that will need to be more deeply understood. We also note that the timescale  $\tau_{grr}$  is long, so all the physics associated with the other time scales represented here must be solved to know the conditions that might be relevant to the production of gravity waves once (if ever) those become the dominant sink.

### 3. The De-leptonization Phase

The contraction phase (Burrows & Lattimer 1986; Keil & Janka 1995; Pons et al. 1999; Villain et al. 2004) will lead to spin up and perhaps to crossing the threshold for NAXI or enhancing the growth rate of these instabilities, the amplitude of which may depend on  $T/|W|$ . These instabilities will cause some loss of rotation energy and angular momentum as the core contracts, perhaps altering the specific non-axisymmetric modes that come into play. The core will dissipate its differential rotation and angular momentum until the loss rates become comparable to, or longer than, the contraction time scale. The de-leptonization phase will depend on details of PNS evolution and we will sketch here the qualitative behavior

to be expected.

We schematically summarize various possible behaviors in Fig. 1, which presents the loci of evolution in the plane representing  $T/|W|$  versus the radius of the contracting PNS core, with time as an implicit parameter. Key values of  $T/|W|$ , 0.27 (incompressible dynamical bar), 0.20 (compressible dynamical bar), 0.14 (secular bar), 0.08 (spiral mode of Ott et al. 2005), and 0.01 (minimum for NAXI) are shown as horizontal dashed lines. We also give on the right axis, an estimate of the effective, mass-weighted rotation period for a given  $T/|W|$ . We note that these periods are only approximate and are, for instance, somewhat longer for a given  $T/|W|$  than those given by Ott et al. (2006).

As indicated by the bold arrow on the upper right boundary of Fig. 1, any PNS born with  $T/|W| \gtrsim 0.14$  is likely to spin down quickly to at least  $T/|W| \sim 0.14$  and probably to significantly smaller values prior to the contraction phase. The radius will, of course, depend on the degree of rotation. If the condition of secular instability to a bar mode is reached by spin-up during contraction, the bar will form, but quickly (in a few hundred ms) dissipate its energy to magnetoacoustic waves. Stability will be reached, but continued contraction driven by neutrino losses will again cause the core to become unstable and spin down. In reality, this will be a continuous process, so that if the dissipation by other NAXI modes does not prevent spin up to this threshold, then during contraction the core should evolve along the locus for secular bar-mode instability,  $T/|W| \sim 0.14$ . The region above  $T/|W| \sim 0.14$  and to the left of the right hand axis is thus a “forbidden region,” which cannot be occupied during the contraction phase.

Thin solid lines in Fig. 1 portray the locus of contraction for constant angular momentum,  $J$ , assuming homologous contraction so that  $T/|W| \sim J^2/GM^3R \propto R^{-1}$ . For comparison, we also show in Fig. 1 the locus of models from Villain et al. (2004) who present a series of stationary rotating models with the thermodynamics set by the evolution of related non-rotating models. The particular models illustrated in Fig. 1 correspond to rigidly-rotating configurations for a PNS of  $1.6 M_\odot$  rotating at breakup. The total angular momentum formally decreases along this sequence. While this is not a fully self-consistent evolutionary sequence of a rotating, de-leptonizing PNS, it does represent a possible path of a contracting PNS that sheds angular momentum. This locus is less steep than  $R^{-1}$ , presumably reflecting the implicit loss of angular momentum.

Schematically, a PNS born with  $T/|W| \sim 0.08$  that contracts with  $T/|W| \propto R^{-1}$  would contact the secular instability line at  $T/|W| \sim 0.14$  and then follow the locus with  $T/|W| \sim 0.14$  as shown by the horizontal arrow as the radius continued to shrink until the contraction finished. Presuming the structure at that point was still unstable to NAXI, the evolution would follow the vertical arrow on the left axis.

Again assuming  $T/|W| \propto R^{-1}$ , we find that a PNS born with  $T/|W| \sim 0.03$  could contract to  $T/|W| \sim 0.14$  at the end of the contraction phase as shown by the middle thin line. Under the same assumption, a PNS born at the lower limit for NAXI,  $T/|W| \sim 0.01$  would contract to  $T/|W| \sim 0.05$ , as shown by the lower thin line. Such a configuration could still be unstable to NAXI even after the contraction phase were complete. Any PNS born with  $T/|W| \gtrsim 0.01 R_{\text{ns}}/R_{\text{pns}} \sim 0.002$  might trigger NAXI before the contraction is complete. Here the subscript “pns” refers to the PNS with radius  $\sim 50$  km and the subscript “ns” refers to the neutron star with radius  $\sim 10$  km.

A more likely evolution than conserving angular momentum would be for the structure to follow the locus defined by the equivalence of the contraction timescale and the time scale for loss of differential rotation free energy to magnetoacoustic waves excited by NAXI. Deducing this locus will require elaborate calculations. Based on our estimate of the timescale for dissipation of the rotational free energy given in Eqn. 4, our best guess is that the dissipation time for NAXI is initially rapid compared to the contraction time. This would suggest an evolution to lower  $T/|W|$  at essentially constant radius, as shown by the lower vertical arrow on the right side of Fig. 1. As the radius decreases, the timescale for dissipation will lengthen. The amplitude of the perturbation is also likely to decrease as  $T/|W|$  declines. As the dissipation time lengthens to be comparable to the contraction time, the locus of evolution will move to the left in Fig. 1. In Fig. 1, we show one possibility, an evolution parallel to, but somewhat above the minimum value of  $T/|W| \sim 0.01$ , the notion again being that if contraction increases  $T/|W|$  too much, the dissipation will increase to keep the timescales approximately comparable.

Once full contraction is achieved, the contracted neutron star is still likely to be spinning faster than any infalling or outgoing material around it, so there could also be a final spin-down phase with the neutron star at constant radius until the inner structure were completely stable to NAXI and other interactions. A rotating neutrino wind might contribute at this epoch (Thompson et al. 2004).

A key question is how much energy can be liberated as magnetoacoustic flux during this contraction phase. Most of the difference in binding energy between the PNS and the final, contracted, cool, neutron star will be emitted in neutrinos (although with steadily smaller luminosity), but for a rotating, magnetized PNS, some of this energy will be dissipated in magnetoacoustic power. In one extreme case corresponding to continued rapid (on the contraction timescale) dissipation by NAXI, the initial rotational energy would be dissipated before contraction would occur. In that case, virtually all the excess binding energy would be lost to neutrinos, and the available rotational energy would just be the initial value of the free energy in differential rotation.



In the opposite extreme case that angular momentum is conserved during the contraction phase,  $J = \text{const}$ , which is equivalent to no dissipation into magnetoacoustic flux, we have after contraction, but before final spin-down:

$$T_{ns} \sim \frac{W_{ns}}{W_{pns}} \frac{R_{pns}}{R_{ns}} T_{pns} \sim \left( \frac{R_{pns}}{R_{ns}} \right)^2 T_{pns}. \quad (6)$$

We have again assumed that  $T/|W|$  and  $W$  both scale as  $R^{-1}$ . In this case, a maximal amount of the excess binding energy is converted to rotational energy after contraction. This condition of constant angular momentum is commonly assumed in contraction of isolated neutron stars (Villain et al. 2004; Ott et al. 2006), but is unlikely to apply to real rotating magnetic PNS still buried within the collapse ambiance unless they are rotating very slowly.

If the PNS evolves by shedding magnetoacoustic energy at roughly constant  $T/|W|$  then

$$T_{ns} \sim \frac{W_{ns}}{W_{pns}} T_{pns} \sim \frac{R_{pns}}{R_{ns}} T_{pns}, \quad (7)$$

and

$$J_{ns} \sim \sqrt{\frac{T_{ns}}{T_{pns}} \frac{R_{ns}}{R_{pns}}} J_{pns} \sim \sqrt{\frac{R_{ns}}{R_{pns}}} J_{pns}. \quad (8)$$

These expressions again assume that the contraction is homologous so that the change in radius is an appropriate measure of the change in binding energy. One example of such possible behavior would be if the PNS were born or quickly evolved to a condition of marginal stability to a secular bar mode and then contraction occurs along the locus of secular bar instability with  $T/|W| \sim 0.14$ . Another example would be contraction along the locus near the lower threshold for exciting NAXI, as illustrated in Fig. 1.

The amount of energy that could be dumped from the rotation to magnetoacoustic energy during the contraction is thus a rather sensitive function of how the dissipation affects the angular momentum. In the extreme case of conserved angular momentum, if the contraction were quasi-homologous so that  $T/|W|$  did scale closely as  $R^{-1}$ , then Eqn. 6 suggests that a fraction  $(R_{pns}/R_{ns})(T_{pns}/|W_{pns}|)$  of the final binding energy, something of order 25 times the initial rotational energy, could be invested in rotational energy and then liberated in spin down magnetoacoustic power. This rotational energy must be limited since, as we argue here, no contracting configuration can penetrate into the forbidden zone at  $T/|W| \gtrsim 0.14$ . As an example, if we assume that the PNS is born with  $T/|W| \sim 0.08$  and contracts along a locus of  $J = \text{const}$  (uppermost thin line in Fig. 1) with negligible production of magnetoacoustic flux until it reaches  $T/|W| \sim 0.14$  and then follows that locus during the remainder of the contraction, the configuration will end up at the end of contraction with  $T/|W| \sim 0.14$  and hence  $T_{ns} \sim 0.70|W_{pns}| \sim 9T_{pns}$ , assuming the radius

has contracted by a factor of 5 and the binding energy has increased proportionately. This energy, still a considerable amount but much less than  $25T_{pns}$ , would then be available in a final spin down phase.

Note that this estimate represents a lower limit to the total amount of magnetoacoustic power liberated because some must be emitted during the contraction along the locus  $T/|W| \sim 0.14$ , the very reason that locus is followed. The virial theorem gives a constraint on the total energy lost to neutrinos and magnetoacoustic flux:

$$\Delta E_\nu + \Delta E_{NAXI} = - \left[ \frac{1 - 2\eta}{3 \langle \gamma - 1 \rangle} + \eta - 1 \right] \Delta W, \quad (9)$$

where  $\eta$  is the ratio of  $T/|W|$  along the locus, assumed to be constant,  $\langle \gamma - 1 \rangle$  is an appropriate average over the adiabatic index, and  $\Delta W$  is the (negative) change in the binding energy. This gives  $\Delta E_\nu + \Delta E_{NAXI} \sim \Delta W \sim W_{ns}$ , but the fraction of the energy lost that goes to neutrinos and that to magnetoacoustic flux would have to be determined by a direct integration of the appropriate luminosity over the contraction time. The loss to neutrinos is likely to dominate the loss to magnetoacoustic flux.

Contraction from  $T/|W| \sim 0.03$  with constant  $J$  (middle thin line in Fig. 1) would, in this illustration, reach  $T/|W| \sim 0.14$  just as contraction was complete. This would give a rotational energy at that point to be dissipated of  $T_{ns} \sim 0.7W_{pns}$ . For initial  $T/|W| \sim 0.01$ , the smallest values in the literature associated with NAXI, the spin energy available to be liberated after contraction if angular momentum were conserved during contraction could be  $\sim 0.05|W_{pns}| \sim 0.25|W_{pns}|$ .

As argued above, the rapid timescale for the dissipation of NAXI (Eqn. 4) suggests that much of the rotational dissipation will occur before contraction. In this case, the initial rotational free energy will be dissipated before any contraction occurs and  $\Delta T \sim T_{pns} \lesssim 0.14W_{pns}$ . As an example, if a PNS is born with  $T/|W| \sim 0.08$ , it will lose rotational energy  $T \sim 0.08W_{pns}$  in prompt spin down. Subsequently, as illustrated by the lower horizontal arrow in Fig. 1, contraction could drive the structure along the locus with  $T/|W| \gtrsim 0.01$  with relatively little more energy in magnetoacoustic flux. The actual evolution is likely to fall between the extremes of conservation of angular momentum during contraction and rapid spin-down prior to contraction.

## 4. Conclusions

The implication is that if a supernova does not succeed on a time scale of  $\sim 100$  ms, de-leptonization is likely to cause the PNS to contract and spin up and amplify the driving

and magnetic dissipation of non-axisymmetric modes that could pump energy into the environment of the neutron star, potentially energizing the explosion if  $T/|W| \gtrsim 0.01$  during the evolution. Even if an explosion has been launched on a time scale of  $\sim 100$  ms, the infall will not have declined to zero in the early part of the contraction process. The ambient density will still give a medium in which to produce and dissipate magnetoacoustic waves generated by non-axisymmetric instabilities.

The contraction phase could thus be important for both the energetics of the explosion and the rotational period of the contracted neutron star, so it is important to understand this phase. We have made some estimates of the energy that could be deposited by magnetoacoustic waves, but caution that this will depend on the actual free energy of differential rotation, the details of the change of structure and hence binding energy during contraction and the mode of the magnetoacoustic flux. If the latter were to emerge up the axis as Poynting flux, it might not contribute to the kinetic energy budget of the expanding ejecta. The contraction could lead to a rapidly rotating neutron star. As we show here, however, a plausible result is a rather rapid spin down prior to substantial contraction followed by contraction at roughly constant  $T/|W| \sim 0.01$  corresponding to the threshold for NAXI. That would correspond to a post-contraction rotational period of  $\sim 3$  ms prior to any final spin down.

The evolution during the de-leptonization phase will depend on the physics and time scales of the dissipation of the free energy of differential rotation. We noted that in our simple model for the time scale associated with the magnetoacoustic flux generated by NAXI, Eqn. 4, the time scale could be rather short compared to the contraction time. Eqn. 4 depends on the rotation frequency implicitly through  $T/|W| \propto \Omega^2$  and  $\delta r/r$ . This is in contrast to the dissipation timescale in a model in which the magnetic field serves to provide a magnetic viscosity (Thompson et al. 2005; Wilson et al. 2005; Ott et al. 2006), for which the dissipation power scales as  $\Omega^3$  and hence the time scale as  $\Omega^{-1}$ . We note that the time scale associated with this process is still model dependent with Thompson et al. (2005) and Ott et al. (2006) adopting a dissipation power per unit mass of  $\sim \alpha h_P^2 \Omega (d\Omega/d\ln r)^2 \sim \alpha h_P^2 \Omega^3$  and hence a time scale of  $\tau_{diss} \sim (\alpha \Omega)^{-1} (r/h_P)^2$ . Wilson et al. (2005) invoke a threshold magnetic field but effectively adopt a dissipation power per unit mass of  $1/2 \Omega^3 r^2$  and hence a time scale of  $\sim \Omega^{-1}$ . While both of these time scales vary as  $\Omega^{-1}$ , that of Wilson et al. (2005) is shorter by a factor of  $\sim (h_P/r)^2$ .

Another important difference between the dissipation by NAXI and magnetoacoustic flux and dissipation by magnetic viscosity is that the magnetic viscosity is, by assumption, dissipated locally as heat, whereas the magnetoacoustic flux may, and probably will, propagate and deposit the dissipated energy non-locally. Deposition near the standing shock,

if one remains, is a likely locale for dissipation, since the magnetoacoustic waves are likely to propagate down the density gradient and unlikely to penetrate the entropy jump at the shock. Determining the fraction of the free energy of differential rotation that is dissipated by magnetic or other forms of viscosity as a local heat source and the fraction radiated and deposited non-locally in magnetoacoustic flux will require detailed calculations.

The contraction phase of a PNS within the context of realistic supernova conditions (successful or not) has not been considered in the literature, but in the context of rotating, magnetic, non-axisymmetric structures, this phase may be critical to understand the energetics of the explosion and the post-supernova state of the neutron star. The contraction phase, however it behaves, will be complete before any neutron star is exposed to external observations.

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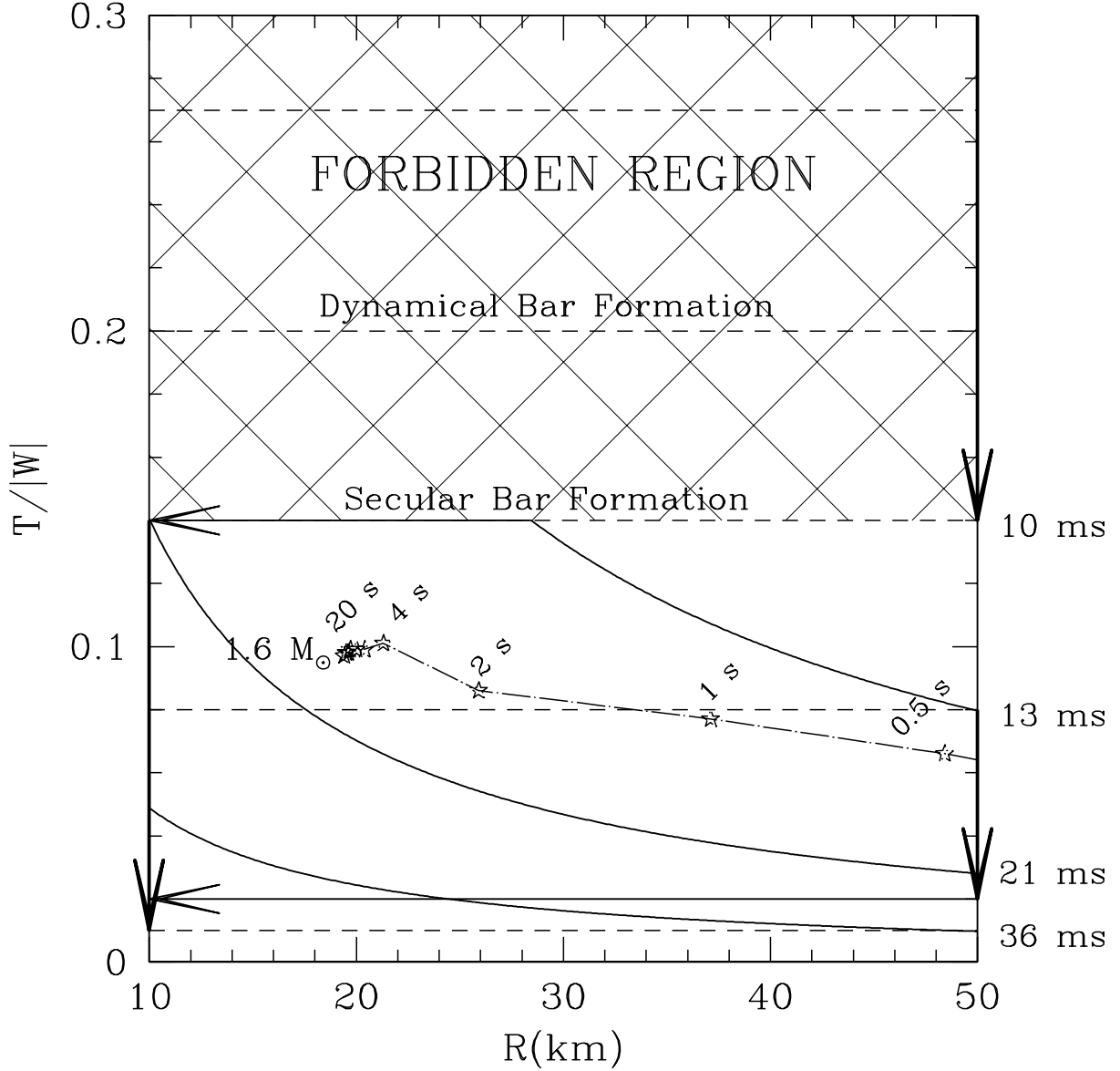


Fig. 1.— Characteristics of the de-leptonization, contraction phase of rotating, magnetic neutron stars are shown in the plane of  $T/|W|$  versus  $R$ . Several key values of  $T/|W|$  representing thresholds for instability (or other representative values) are given by dashed horizontal lines. The curved lines represent homologous contraction conserving angular momentum. Arrows represent possible loci of contraction under the influence of non-axisymmetric instabilities and radiation of magnetoacoustic flux (see text for details). The curve denoted by time marks is taken from Villain et al. (2004). The right-hand axis gives an estimate of the rotational period for a given value of  $T/|W|$  based on a simple model with constant density within the inner solar mass.

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